



LAWRENCE  
LIVERMORE  
NATIONAL  
LABORATORY

# Final Report - Summer Visit 2010

R. Bank

September 14, 2011

## **Disclaimer**

---

This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.



To: Rob Falgout  
From: Randy Bank  
Subject: Summer Visit  
Date: August 24, 2011

## Final Report—Summer Visit 2010

During my visit to LLNL during the summer of 2010, I worked on algebraic multilevel solvers for large sparse systems of linear equations arising from discretizations of partial differential equations. The particular solver of interest is based on ILU decomposition. The *setup* phase for this AMG solve is just the single ILU decomposition, and its corresponding error matrix. Because the ILU uses a minimum degree or similar sparse matrix ordering, most of the fill-in, and hence most of the error, is concentrated in the lower right corner of the factored matrix.

All of the major multigrid components – the smoother, the coarse level correction matrices, and the fine-to-coarse and coarse-to-fine rectangular transfer matrices, are defined in terms of various blocks of the ILU factorization. Although such a strategy is not likely to be optimal in terms of convergence properties, it has a relatively low setup cost, and therefore is useful in situations where setup costs for more traditional AMG approaches cannot be amortized over the solution of many linear systems using the same matrix. Such a situation arises in adaptive methods, where often just one linear system is solved at each step of an adaptive feedback loop, or in solving nonlinear equations by approximate Newton methods, where the approximate Jacobian might change substantially from iteration to iteration.

In general terms, coarse levels are defined in terms of successively smaller lower right blocks of the matrix, typically decreasing geometrically in order. The most difficult issue was the coarse grid correction matrix. The preconditioner/smoothener for a given level is just the corresponding lower right blocks of the ILU factorization. The coarse level matrix itself is just the Schur complement; this matrix is not known exactly using just the ILU decomposition in the setup phase. Thus we approximate this matrix using various combinations of the preconditioning matrix and the error matrix. During my visit, several approximations of this type were implemented and tested. While some improved the convergence rate of the overall method, these gains had to be balanced against the additional costs involved in creating and applying these matrices. By this more stringent criterion, none of the improved approximations could be characterized as an unqualified success.